PVP 23

Code: 23BS1101

I B.Tech - I Semester – Regular Examinations - JANUARY 2024

LINEAR ALGEBRA & CALCULUS
(Common for ALL BRANCHES)

Duration: 3 hours | Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level | CO – Course Outcome

PART – A

		BL	CO
1.a)	Estimate the value of a , if the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\0 & a & 4\\1 & -1 & 1\end{bmatrix}$ is 2	L2	CO1
1.b)	If the initial approximation to the solution of $10x+2y+z=9,\ 2x+20y-2z=-44,\ -2x+3y+10z=22$ is $(x,y,z)=(0,0,0)$ then find the first approximation by using Gauss-Seidel iteration method.	L3	C04
1.c)	If the eigen values of $A=\begin{bmatrix}3 & -1 & 1\\-1 & 5 & -1\\1 & -1 & 3\end{bmatrix}$ are 2, 3 & 6 then predict the eigen values of A^{-1}.	L2	CO2
1.d)	Write down the quadratic form X^TAX corresponding to the symmetric matrix $A=\begin{bmatrix}1 & 3 & -5\\3 & 2 & 0\\-5 & 0 & -4\end{bmatrix}$	L2	CO4
1.e)	Discuss the applicability of Cauchy's mean value theorem for $f(x)=\begin{cases}-x, & if\ -4<x<0\\ x, & if\ 0\le x<4\end{cases}$ and $g(x)=x^2$ in $[-4,4]$	L2	CO3

1.f)	State the Maclaurin's series expansion of $f(x)$ about $x = 0$.	L1	CO3
1.g)	Estimate $\lim\limits_{\substack{x\to 1\\ y\to 2}} \frac{2x^2y}{x^2+y^2+1}$	L2	CO1
1.h)	Estimate the first and second order partial derivatives of $f(x,y)=ax^2+2hxy+by^2$	L2	CO1
1.i)	Write the limits by changing the order of integration of the double integral $\int_0^1\int_y^{y^2}(x+y)\ dxdy$ with the help of region of integration.	L2	CO5
1.j)	Calculate the double integral $\int_0^1\int_0^1 xy\ dydx$.	L3	CO5

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Discover the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$ by reducing the matrix to Echelon form.	L3	CO2	5 M
	b)	Solve the system of non-homogeneous linear equations $5x_1+3x_2+7x_3=4$, $3x_1+26x_2+2x_3=9$ and $7x_1+2x_2+10x_3=5$	L3	CO2	5 M
		OR			
3	a)	Apply Gauss Jordan method to find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$	L3	CO2	5 M
	b)	Make use of Jacobi's method to find first five iterations of the following system of equations $20x+y-2z=17$, $3x+20y-z=-18$, $2x-3y+20z=25$	L3	CO2	5 M

		UNIT-II			
4	a)	Calculate the characteristic roots and characteristic vectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	L3	CO2	5 M
	b)	Make use of the eigen values of matrix of the quadratic form to discuss the rank and nature of the quadratic form $-x_1^2-4x_2^2-x_3^2+4x_1x_2-4x_2x_3-2x_1x_3$	L4	CO4	5 M
		OR			
5	a)	Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find A^4.	L3	CO2	5 M
	b)	Use Diagonalization to find the matrix *A*, if the eigen values of a matrix A of order 3 and the corresponding eigen vectors are 0,3,15 & $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ respectively.	L3	CO2	5 M
		UNIT-III			
6	a)	Check the applicability of Rolle's theorem, if applicable verify theorem for the function $\log\left\{\frac{x^2+ab}{x(a+b)}\right\}$ in $[a,b]$, where $0<a<b$	L3	CO5	5 M
	b)	Construct the series expansion of $f(x)=\log(1+x)$ in powers of x up to third degree terms.	L3	CO5	5 M
		OR			
7	a)	Apply mean value theorem to prove that $\frac{b-a}{1+b^2} < tan^{-1} b - tan^{-1} a < \frac{b-a}{1+a^2}$ $(0 < a < b)$ and hence deduce that $\frac{\pi}{4}+\frac{3}{25} < tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$.	L3	CO5	5 M

	b)	Discover the series expansion of $f(x) = Sin\,x$ in powers of $x - \frac{\pi}{4}$	L3	CO5	5 M
UNIT-IV					
8	a)	Point out the functions $u = x\,e^y sin\,z,\;\; v = x\,e^y cos\,z,\;\; w = x^2 e^{2y}$ are functionally dependent or not. If functionally dependent, find the relation between them.	L3	CO5	5 M
	b)	Discover the nature of stationary points and then find extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$	L3	CO3	5 M
OR					
9	a)	Make use of functional determinant to show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3 \sin 2\theta$ where $u = x^2 - 2y^2, v = 2x^2 - y^2$ and $x = r\cos\theta, y = r\sin\theta$	L3	CO5	5 M
	b)	Divide twenty-four into three parts such that the continued product of the first part, square of the second part and the cube of third part is maximum.	L4	CO3	5 M
UNIT-V					
10	a)	By changing the order of integration, evaluate the double integral $\int_0^2 \int_{e^x}^{e} \frac{1}{log\,y}\,dy\,dx$	L3	CO5	5 M
	b)	Calculate the volume of the solid bounded by the planes $x = 0, y = 0, z = 0\ and\ x + y + z = 1.$	L3	CO3	5 M
OR					
11	a)	Calculate the triple integral $\int_{-1}^{1} \int_0^2 \int_1^3 x^2y^2z^3\,dx\,dy\,dz.$	L3	CO5	5 M
	b)	Discover the area enclosed by the pair of curves $y^2 = x$ and $y = x^2$ using double integration.	L3	CO3	5 M

I B.Tech – I Sem – Reg – January 2024

Linear Algebra & Calculus:

Scheme of evaluation:

1 (a) $R_3 \to R_3 - R_1 \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & a & 4 \\ 0 & -3 & -2 \end{bmatrix}$; $R_3 \to R_3 a + 3R_2 \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & a & 4 \\ 0 & 0 & -2a+12 \end{bmatrix}$ — 1M

$\because \rho(A) = 2 \quad \therefore a = 6.$ — 1M

1 (b)
$$\left.\begin{aligned} x &= \frac{1}{10}[9 - 2y - z] \\ y &= \frac{1}{20}[-44 - 2x + 2z] \\ z &= \frac{1}{10}[22 + 2x - 3y] \end{aligned}\right\} \longrightarrow 1M$$

I st appr:

$x_1 = 0.9$, $y_1 = -2.29$, $z_1 = 3.067$

$$\left.\begin{aligned} x &= 9/10 \\ y &= -4\cancel{5}8/20 \\ z &= \cancel{22/10}\ \frac{30.67}{10} \end{aligned}\right\} \to 1M$$

1 (c) The eigen values of A^{-1} are $\frac{1}{2}, 1/3, 1/6 \to$ 2M

1 (d) $x^T A x = [x_1\ x_2\ x_3] \begin{bmatrix} 1 & 3 & -5 \\ 3 & 2 & 0 \\ -5 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \to$ 1M

$= x_1^2 + 2x_2^2 - 4x_3^2 + 6x_1x_2 - 10x_1x_3 \to$ 1M

1 (e) $Lf'(0) = -1$; $Rf'(0) = 1 \longrightarrow$ 1M

$\therefore f(x)$ is not derivable at $x = 0$

Hence Cauchy's Th is not applicable — 1M

1 (f) $f(x) = f(0) + x f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \cdots + \frac{x^n}{\lfloor n} f^n(0) + \cdots \to$ 2M

1 (g) $\lim_{y \to 2} \frac{2y}{y^2+2}$ — 1M $= \frac{2(2)}{6} = 2/3$ — 1M

1 (h)
$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 2ax + 2hy; \quad \frac{\partial^2 f}{\partial x^2} = 2a \\ \frac{\partial f}{\partial y} &= 2hx + 2by; \quad \frac{\partial^2 f}{\partial y^2} = 2b \end{aligned}\right\} \to 1M \qquad \frac{\partial^2 f}{\partial x \partial y} = 2h \to 1M$$

1 (i) $x \to 0$ to 1 — 1M

$y \to \sqrt{x}$ to $x \to$ 1M

1 (j) $\frac{1}{2}\int_0^1 x\,dx$ — 1M

$= 1/4$ — 1M

PART-B.

2 (a) $R_2 \to R_2 + R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & -4 & -2 \\ 0 & -1 & 2 & 1 \\ 0 & -2 & 4 & 2 \end{bmatrix} \to (2M)$$

$R_3 \to 2R_3 + R_2$, $R_4 \to R_4 + R_2$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & -4 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to (2M)$$

$\therefore \rho(A) = 2$ —— (1M)

2 (b) The augmented matrix $C = [A|B]$

$$C = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right] \to (1M)$$

$R_2 \to 5R_2 - 3R_1$, $R_3 \to 5R_3 - 7R_1$ $\sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{array}\right]$

$R_2 \to R_2/11$; $R_3 \to R_3 + R_2$ $\sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$r = \rho(A) = \rho(C) = 2 \to (2M)$: $n = 3$

$r < n$: So its solutions are infinite

$5x + 3y + 7z = 4$

$11y - z = 3$

put $z = k$ then $y = 3/11 + k/11$; $x = -16k/\cancel{8}11 + 7/11 \to (2M)$

3 (a) $A = IA$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 4R_1$ $\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -8 & -8 \\ 0 & -6 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} A \to (2M)$

$R_1 \to 4R_1 + R_2$, $R_3 \to 8R_3 - 6R_2$ $\sim \begin{bmatrix} 4 & 0 & 4 \\ 0 & -8 & -8 \\ 0 & 0 & -40 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -3 & 1 & 0 \\ -14 & -6 & 8 \end{bmatrix} A$

$R_1 \to 10R_1 + R_3$ $\sim \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & -40 \end{bmatrix} = \begin{bmatrix} -4 & 4 & 8 \\ -1 & 11 & -8 \\ -14 & -6 & 8 \end{bmatrix} A \to (2M)$

$R_2 \to 5R_2 - R_3$

$R_1 \to R_1/40$; $R_2 \to R_2/40$; $R_3 \to R_3/-40$

we get $A^{-1} = \begin{bmatrix} -1/10 & 1/10 & 1/5 \\ 1/40 & -11/40 & 1/5 \\ 7/20 & 3/20 & -1/5 \end{bmatrix} \to (1M)$

3 (b) $x = \frac{1}{12}[17 - y + 2z]$; $y = \frac{1}{20}[-18 - 3x + z]$; $z = \frac{1}{20}[25 - 2x + 3y]$ $\longrightarrow$ (1M)

let $x_0 = 0$; $y_0 = 0$; $z_0 = 0$.

Ist approximation: $x_1 = 0.85$, $y_1 = -0.9$; $z_1 = 1.25$

2nd appr " $x_2 = 1.02$; $y_2 = -0.965$; $z_2 = 1.03$.

3rd appr " $x_3 = 1.00125$; $y_2 = -1.0015$; $z_3 = 1.0025$

Similarly: 5th approximation $x = 1$; $y = -1$; $z = 1$ $\longrightarrow$ (2M)

4 (a) The characteristic eq of A is $|A - \lambda I| = 0$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$$

$$\Longrightarrow \lambda = 0, 3, 15 \longrightarrow \text{(2M)}$$

for

$\lambda = 0$ corresponding eigen vector is $x_1 = \begin{bmatrix} k \\ 2k \\ 2k \end{bmatrix}$

$\lambda = 3$ " " $x_2 = \begin{bmatrix} 2l \\ l \\ -2l \end{bmatrix}$

$\lambda = 15$ " " $x_3 = \begin{bmatrix} 2m \\ -2m \\ m \end{bmatrix}$ $\longrightarrow$ (3M)

4 (b) The symmetric matrix $A = \begin{bmatrix} -1 & 2 & -1 \\ 2 & -4 & -2 \\ -1 & -2 & -1 \end{bmatrix}$ $\longrightarrow$ (1M)

The characteristic eq of A is $|A - \lambda I| = 0$

$-\lambda^3 - 6\lambda^2 + 16 = 0 \Rightarrow (\lambda - 2)(\lambda^2 + 4\lambda - 8) = 0 \longrightarrow$ (1M)

$\therefore \lambda = -2$, $\lambda = \frac{-4 \pm \sqrt{48}}{2} = -1 \pm \sqrt{12}$ $\longrightarrow$ (1M)

Rank = 3 ; nature is indefinite. $\longrightarrow$ (2M)

5 (a) The characteristic eq of A is $\lambda^3 - 3\lambda^2 - \lambda + 9 = 0$

by Cayley Hamilton th. every square matrix satisfies its own characteristic equation.

$$\therefore A^3 - 3A^2 - A + 9I = 0 \longrightarrow \text{(1M)}$$

$A^2 = \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix}$; $A^3 = \begin{bmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{bmatrix}$ $\longrightarrow$ (1M)

$$A^3 - 3A^2 - A + 9I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow 1m$$

$$A^4 = 3A^3 + A^2 - 9A$$

$$= \begin{bmatrix} 7 & -30 & 42 \\ 18 & -13 & 46 \\ -8 & -14 & 17 \end{bmatrix} \longrightarrow 2m$$

5(b) P = grouping of eigenvectors:

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \longrightarrow 1m$$

$$P^{-1} = \frac{\text{adj}\, P}{\det P} = \begin{bmatrix} 1/9 & 2/9 & 2/9 \\ 2/9 & 1/9 & -2/9 \\ 2/9 & -2/9 & 1/9 \end{bmatrix} \longrightarrow 2m$$

$$A = P \cdot D \cdot P^{-1} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \longrightarrow 2m$$

6(a) Let $f(x) = \log(x^2+ab) - \log x - \log(a+b)$ $\quad 0<a<b$

$f(x)$ is continuous in $[a,b]$ & derivable in (a,b); $f(a) = \log\left(\frac{a^2+ab}{a^2+ab}\right) = 0$

$f'(x) = \frac{x^2-ab}{x(x^2+ab)} \longrightarrow 2m$; $\quad f(b) = \log\left(\frac{b^2+ab}{ab+b^2}\right) = 0.$

$\therefore f(a) = f(b)$

all the three conditions of Rolle's th. are satisfied.

$\therefore \exists$ a pt $c \in (a,b) \ni f'(c) = 0$

$\Rightarrow \frac{c^2-ab}{c(c^2+ab)} = 0 \Rightarrow c = \pm\sqrt{ab} \longrightarrow 2m$

$c = \sqrt{ab} \in (a,b)$. Hence Rolle's th. is verified $\longrightarrow 1m$

6(b)

$$\left.\begin{aligned} f(x) &= \log(1+x) & f(0) &= 0 \\ f'(x) &= (1+x)^{-1} & f'(0) &= 1 \\ f''(x) &= -1(1+x)^{-2} & f''(0) &= -1 \\ f'''(x) &= 2(1+x)^{-3} & f'''(0) &= 2 \end{aligned}\right\} \longrightarrow 2m$$

$\therefore f(x) = f(0) + x f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \cdots \longrightarrow$ (1M)

$= x - x^2/2 + x^3/3 - \cdots \longrightarrow$ (2M)

7 (a) Consider $f(x) = \tan^{-1}(x)$

$f(x)$ is continuous in $[a,b]$ and derivable in (a,b)

By Lagranges mean value Th. $\exists$ a pt $c \in (a,b) \ni f'(c) = \frac{f(b)-f(a)}{b-a} \longrightarrow$ (1M)

$f'(x) = \frac{1}{1+x^2}$; $f'(c) = \frac{1}{1+c^2}$

$\exists$ a pt c, $a < c < b \ni \frac{1}{1+c^2} = \frac{\tan^{-1} b - \tan^{-1} a}{b-a}$

$1+a^2 < 1+c^2 < 1+b^2$

$\Rightarrow \frac{1}{1+a^2} > \frac{1}{1+c^2} > \frac{1}{1+b^2}$

$\frac{1}{1+a^2} > \frac{\tan^{-1} b - \tan^{-1} a}{b-a} > \frac{1}{1+b^2}$

or $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2} \longrightarrow$ (2M)

put $a = 1$; $b = 4/3$.

$3/25 < \tan^{-1} 4/3 - \pi/4 < 1/6$

$3/25 + \pi/4 < \tan^{-1} 4/3 < \pi/4 + 1/6 \longrightarrow$ (2M)

7(b) $f(x) = \sin x$; $a = \pi/4$

$f(\pi/4) = 1/\sqrt{2}$

$f'(x) = \cos x$; $f'(\pi/4) = 1/\sqrt{2}$

$f''(x) = -\sin x$; $f''(\pi/4) = -1/\sqrt{2}$

$f'''(x) = -\cos x$; $f'''(\pi/4) = -1/\sqrt{2}$

$f^{IV}(x) = \sin x$; $f^{IV}(\pi/4) = 1/\sqrt{2}$ $\longrightarrow$ (2M)

required series expansion $\longrightarrow$ (1M)

$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{\lfloor 2} f''(a) + \cdots$

$\therefore \sin x = 1/\sqrt{2} + (x - \pi/4) 1/\sqrt{2} - \frac{(x-\pi/4)^2}{\lfloor 2} \cdot 1/\sqrt{2}$

$- \frac{(x-\pi/4)^3}{\lfloor 3} \cdot 1/\sqrt{2} + \cdots$ $\longrightarrow$ (2M)

8 (a)

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} e^y \sin z & x e^y \sin z & x e^y \cos z \\ e^y \cos z & x e^y \cos z & -x e^y \sin z \\ 2x e^{2y} & 2x^2 e^{2y} & 0 \end{vmatrix} \longrightarrow (2m)$$

expending the above determinant

$$= 2x^3 e^{4y} \cancel{\sin^2 z} - 2x^3 e^{4y} \cancel{\sin^2 z}$$

$$= 0$$

$\therefore u, v, w$ are functionally related $\longrightarrow$ (2m)

Relation is $w = u^2 + v^2 \longrightarrow$ (1m)

8 (b) $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

$\frac{\partial f}{\partial x} = 0 \Rightarrow 3x^2 + 3y^2 - 30x + 72 = 0$.

$\frac{\partial f}{\partial y} = 0 \Rightarrow 6xy - 30y = 0$. $\quad 3x^2 - 30x + 72 = 0$.

$\therefore y = 0; x = 5 \quad \therefore x = 4, 6$

Stationary pts are $(4,0), \cancel{(5,0)}, (5,1), (5,-1), (6,0) \longrightarrow$ (1m)

$\therefore r = \frac{\partial^2 f}{\partial x^2} = 6x - 30;\ s = \frac{\partial^2 f}{\partial x \partial y} = 6y;\ t = \frac{\partial^2 f}{\partial y^2} = 6x - 30 \longrightarrow$ (1m)

at $(4,0)$ $r = -6;\ s = 0;\ t = -6$.

$rt - s^2 = 36 > 0;\ r = -6 < 0$.

$\therefore f(x,y)$ is max at $(4,0)$. $\longrightarrow$ (1m)

at $(6,0)$: $r = 6;\ s = 0;\ t = 6$

$rt - s^2 = 36 > 0$ & $r = 6 > 0$.

$\therefore f(x,y)$ is minimum at $(6,0) \longrightarrow$ (1m)

at $(5,0)$ $r = 0;\ s = 0;\ t = 0$.

$\therefore rt - s^2 = 0$: (neither max nor minima)

no conclusion. $\longrightarrow$ (1m)

9 (a) $$\frac{\partial(u,v)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(r,\theta)} \longrightarrow (1m)$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2x & -4y \\ 4x & -2y \end{vmatrix} = 12xy$$
$$= 12 r^2 \sin\theta\cos\theta$$
$$= 6r^2 \sin 2\theta \longrightarrow (2m)$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r \longrightarrow (1m)$$

$$\therefore \frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3 \sin 2\theta \longrightarrow (1m)$$

9 (b) Form the Lagrangian function

$$F(x,y,z) = f(x,y,z) + \lambda\phi(x,y,z)$$
$$= z^3 y^2 x + \lambda(x+y+z-24) \longrightarrow (1m)$$

$$\left.\begin{array}{l} \frac{\partial F}{\partial x} = 0 \Rightarrow -\lambda = z^3 y^2 \\ \frac{\partial F}{\partial y} = 0 \Rightarrow -\lambda = z^3\, 2yx \\ \frac{\partial F}{\partial z} = 0 \Rightarrow -\lambda = 3z^2 y^2 x \end{array}\right\} \longrightarrow (2m)$$

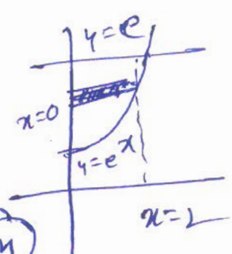
$$\therefore z^3 y^2 = z^3\, 2xy = 3z^2 y^2 x$$

from this we get $y = 2x$; $2z = y$: $x = z$.

$$x+y+z = 24 \Rightarrow 4x = 24 \Rightarrow x = 6.$$
$$\therefore z = 6 ;\ y = 12$$

$$\therefore \text{Max } f(x,y,z) = z^3 y^2 x.$$
$$= 6^3 \cdot 12^2 \cdot 6. \longrightarrow (2m)$$

10 (a) $$\int_0^2 \int_{e^x}^{e} \frac{1}{\log y}\, dy\, dx = \int_{y=\cancel{2}1}^{e} \int_{x=0}^{\log y} \frac{1}{\log y}\, dx\, dy \longrightarrow (2m)$$



by changing the order of integration.

$$= \int_{y=\cancel{2}1}^{e} \frac{1}{\log y} (x)_0^{\log y}\, dy = \int_{y=2}^{e} 1\, dy \longrightarrow (1m)$$

y=1

$$= (y)_1^e = \frac{e-1}{\cancel{e-2}} \quad (2m)$$

$$\boxed{e-1}$$

10 (b)

$$\iiint_V dz\,dy\,dx = \int_{x=0}^{1}\int_{y=0}^{1-x}\int_{z=0}^{1-x-y} dz\,dy\,dx \longrightarrow 1m$$

$$= \int_{x=0}^{1}\int_{y=0}^{1-x}(1-x-y)\,dy\,dx \longrightarrow 1m$$

$$= \int_{x=0}^{1}\left(y - xy - \frac{y^2}{2}\right)_{y=0}^{1-x} dx$$

$$= \int_{x=0}^{1}\left(\tfrac{1}{2}x^2 - x + \tfrac{1}{2}\right)dx \longrightarrow 2m$$

$$= 1/6 \longrightarrow 1m$$

11 (a)

$$\int_{-1}^{1}\int_{0}^{2} y^2 z^3 \left(\frac{26}{3}\right) dy\,dz \longrightarrow 2m$$

$$= \frac{26}{3}\int_{-1}^{1} z^3 \left(\frac{y^3}{3}\right)_0^2 dz$$

$$= \frac{26}{3} \times \frac{8}{3} \times \left(\frac{z^4}{4}\right)_{-1}^{1} \longrightarrow 2m$$

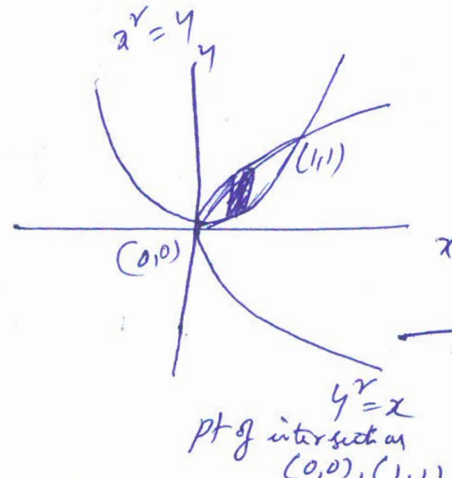
$$= 0 \longrightarrow 1m$$

11 (b)

$x^2 = y$; $y^2 = x$

$x^4 = x \Rightarrow x(x^3-1) = 0$

$\Rightarrow x = 0$; $x = 1 \longrightarrow 1m$



$\longrightarrow 1m$

pt of intersection $(0,0), (1,1)$

$\therefore$ required area is $\displaystyle\int_{x=0}^{1}\int_{y=x^2}^{\sqrt{x}} dy\,dx$

$$= \int_{x=0}^{1}(y)_{x^2}^{\sqrt{x}}\,dx = \int_{x=0}^{1}(\sqrt{x} - x^2)\,dx \longrightarrow 2m$$

$$= \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right)_0^1 = \underline{1/3} \longrightarrow 1m$$